ANALYZING THE POSSIBLE LINE CHOICES

A SKI RACER HAS IN A COURSE

An Interdisciplinary Qualifying Project

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Abstract

This project sought to determine the relationship between time, speed, and the turning radius of a ski racer. This is important because the further development of the knowledge in this field may lead performance enhancement as well as injury prevention. A code was written using Matlab, which could analyze a line composed of a series of data points, to determine the speed and time that the ski racer would spend on the course. Through this research it was found that there is an optimal radius to minimize the time that a ski racer spends on the course and a different optimal radius to maximize the speed of the racer. These optimal radii vary with different speeds, skill levels, course settings, and slope angle.

Foreword

Several years ago by Dr. Brown and Tiffany Lufkin began working on different versions of this problem. They worked out the relationship between turning radius and course time at a constant skier speed. I became involved with the project after Dr. Brown asked for assistance in calculating the time when accelerations are involved. The multi-member research team slowly dwindled to me as the sole active researcher. Interesting results were obtained after several months of work, prompting Dr. Brown and I to explore the option of working with other individuals conducting research in the same field.

The individual doing research most closely resembling mine at the time was Jörg Spörri, a doctoral student at the University of Salzburg, and a former member of the Swiss Europa Cup team, while he was a MS student at ETHZ. Through emails it became apparent that the research that we were doing could compliment each other's efforts, therefore, my travel and stay for two months in Austria was arranged. I presented my findings at the 5th International Congress of Science and Skiing at the end of this stay.

While in Austria I had the opportunity to work with Jörg on his current research. I looked at data that was collected by Jörg and the team at the University of Salzburg to try to develop a model which would predict the speed and time of a skier through a turn of a GS race. The model incorporated all of the movements of the skier including COM movement as well as skid angle. The model is still being worked on and will hopefully shed some light the fastest line down a course as well as how to prevent injuries in the sport of ski racing.

I would like to thank Tiffany Lufkin for her initial work with the project, which provided a base I was able to build upon, and Dr. Brown for providing the opportunity to work in an area of study that I have a passion for. I would like to also thank Jörg Spörri, Erich Müller, all of the individuals working in the Christian Doppler Laboratory, and the Universität Salzburg for the opportunity to work and stay in the beautiful country of Austria.

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Introduction

Objective

The objective of this project is to contribute to the understanding of how line affects the time and speed of a ski racer on a given course.

Rationale

Ski Racing is a sport where a race can be decided by a time difference as small as one hundredth of a second over two runs over one minute each (FIS 2010). That is smaller than %0.008 of the total time that the skier was in the course. With such a small difference between victory and defeat the smallest advantage for one skier could propel him past all of his competition. By understanding the physical principals that correlate to a faster run the ski race may gain a better idea of how to beat his opponent.

Furthering the knowledge of how skiing time and speed with regards to course setting and line skied also has implications outside of simply learning how to gain the advantage over one's opponent. Since ski racing became a competitive sport, the International Skiing Federation (FIS) has been creating rules and regulations to keep the skier safe as he participates in the sport. By understanding what causes skiers to ski as fast as they do the FIS may be able to create regulations based on this information to limit the speed that a skier travels. These rules can either be based on the course setting or on the equipment that the skiers are allowed to use. Recently the FIS increased the turning radius limit from 21m to 27m on the skis that competitors are allowed to use (FIS 2010). This was thought to limit the speed that a skier would be able to travel by forcing the skier to skid more of the turn, but has not eliminated as much injury as they had hoped. The FIS also decreased the height that the ski boot sole is allowed to be above the running surface of the ski from 55mm to 50mm (FIS 2010).

The FIS is currently funding a large injury prevention study at the University of Salzburg. Researchers there are mapping the entire course, including gates and topography, with a GPS device. Then they are putting accelerometers and a GPS on the forerunners to map their movements through the

course. The athletes have to fill out a questionnaire after the race asking them what parts of the course they felt were dangerous and how the safety can be improved. This is the largest injury study the FIS has ever conducted and they hope to make large gains in injury prevention with its findings.

Ski equipment companies may also find information on how speed and time are affected by course setting and line. Atomic sponsors many scientific studies in the skiing world. They are one of the largest partners of the University of Salzburg and work closely with Erich Müller on his studies. They also sponsored the 2010 ICSS conference where all of the top researchers in world met to discuss their research in skiing. With the information they have gained from doing this they may be better able to produce winning equipment for their skiers.

State of the Art

There has been much speculation in what line would be faster for the ski racer to take. A portion of this speculation is based on scientific principal such as the brachistochrone problem, some results from just observation. There are several examples of this work finding its way into books and papers. David Lind and Scott P. Sanders worked on the physical forces that act on a skier. Though this book covers a wide range of skiing topics the authors go into the forces acting on the skier in depth as well as include the brachistochrone problem in a technote (Lind and Sanders, 2004). Ron Lemaster also has written a book describing the optimal technique when making carved turns. He breaks up the turn into four sections, the initiation, control, completion, and transition phase (Lemaster 1999). This four part model is still the most accepted one and is used by both Robert Reid as well as Jörg Spörri in their doctoral work. Tiffany Lufkin and Chris Brown, at Worcester Polytechnic Institute, determined the time that it would take to ski a series of three gates at varying turn radii (presentation ISSS 2009). This was, however, at a constant speed without taking into account that the smaller one's turning radius the more energy they are likely to dissipate. Currently, the lines being taken by ski racers are being measured and analyzed by both Rob Reid (ICSS, dissertation, 2010) in slalom, and Jörg Spörri (ISCC and MS thesis, 2010) in GS. In

these two studies a ski hill was closed off to the public and skiers were sent through a race course while their movements were captured with cameras and force meters. Though these projects were not directly aimed at determining the line that produces the fastest time, this might become a side effect of their studies.

The problem with the brachistochrone line that theoretically should be the fastest between two gates is that it is difficult to ski in such a manner as it involves a sharp direction change when one approaches each gate. Observing skiers one can see that in the technical events of slalom and giant slalom, no one skies in such a manner.

Approach

I constructed a numerical model in Matlab to analyze a given number of lines through a course. The model takes in a given set of data, which is user defined, and analyzes the skier skiing down the prescribed path. But in order to create a working model and analyze the final data, several sub-objectives had to be met to reach the final goal, these were as follows.

- Establishing all of the forces that the skier will encounter that are relevant to the project

- Creation of a realistic course in which the theoretical skier will ski

- Determining how the line that the skier skis will be defined

- Establishing a method for the collection of the data that will compose the line

- Creating a numerical model which will output parameters such as time, speed, and distance for the specific lines the skier will ski

- Determining what entrance speeds to conduct analysis at

- Displaying the results in a manner easily read and analyzed by others

Though this research started in the United States, there is little opportunity for ski research in the United States. There are a few individuals studying alpine ski racing performance in the United States,

one being Dr. David Bacharach from Saint Cloud State University, another is Dr. Christopher Brown from Worcester Polytechnic Institute. Most of the research being done in the United States is conducted privately by the United States Ski and Snowboard Association at their Center of Excellence. In other countries, such as Austria, skiing comprises a much larger part of their economy so more research takes place there. I traveled to Austria to study at the Christian Doppler Laboratory at the University of Salzburg, headed by the esteemed Dr. Müller in order to gain a better understanding of how ski research is conducted in an area where it is more of a priority than in the United States.

Background

Current Research

In 2010 Robert Reid defended his doctoral dissertation on the kinetic and kinematic study of skiing technique in slalom. He used panning and tilting cameras to capture the movements of the skiers so that they could be recorded and transferred into data files. He studied many things about how the skiers skied including the lean angle, fore aft position, and skid angle of the skis. He also calculated the energy dissipated by the skier versus many different variables, including turning radius (Reid 2010).

Jörg Spörri is currently working on his doctoral dissertation at the University of Salzburg with Erich Müller in the Christian Doppler Laboratory. His research is similar to that of Robert Reid's except that he is working mostly with Giant Slalom. However, Jörg is not only focusing on the movements of the skier but also the placement of the line and how it effects the time of the turn. He has found that the higher a turn is initiated and the higher it is ended the faster the turn will be. He found that in these turns the skier spend less energy skidding the top of the turn and was able to initiate the carve earlier (Spörri 2010).

Ski Research in the United States

Ski research in the United States is done mostly at the observatory level. Coaches on the hill will observe skiers, skiing in a variety of situations, and try to determine what separates the better skiers from the ones that aren't as proficient. Often times this is purely hypothetical because there is no concrete reliable data to quantify their theories.

There are several individuals conducting ski research at the institutional level. One such person is Dr. David Bacharach, who works in the Human Performance Laboratory at Saint Cloud University. Since Dr. Bacharach has a PhD in human physiology most of his research is on the performance of the human body in athletic situations, such as dietary needs of a skier while training. He also works on training techniques designed to maximize the efficiency of a training regime in the time an athlete has available.

At the 2010 ICSS conference he presented his research on the effects of forced carbohydrate consumption over a two week training period (Bacharach 2010).

Dr. Christopher Brown is a mechanical engineering professor at Worcester Polytechnic Institute. As an engineer, naturally, most of the research that he does is equipment based, from new types of bindings to ski construction. He also works on the theory of ski racing and how to ski the fastest course possible. This project falls into the theoretical inquiry category of ski racing. For most of these projects there is little funding or grants available, even though through this type of research great strides can be made in the sport of ski racing and skiing in general. Together with Ron Lemaster Dr. Brown presented his research on Euler's equations in skiing at the 2010 ICSS conference. Using Euler's Equations they were able to explain the three dimensional rotations of a skier as he completes a race course (Brown and Lemaster, 2010).

The United States Ski and Snowboard Association have recently completed a new building for the training and research of skiing athletes called the Center of Excellence. It is at this center that much of the research for the United States Ski Team is performed. I asked Dr. Troy Flanagan, who is the current high performance director of USSA but prior to the 2010 Olympics was the director of sports science, about the research that is being conducted for the United States Ski Team. From my interview with Dr. Flanagan, I was able to gain a perspective on what the US is doing to help their top athletes perform at the highest level. The research being done by the USSA is mostly in the disciplines of Alpine and Nordic combined events concentrating on physiology, nutrition, and equipment. The majority of the research done by the USSA is conducted at their internal sport science department but there have been instances where the USSA has partnered with other universities. As far as funding that the USSA receives for this research, which totals around 1 million dollars per Olympic cycle, they receive most of it from grants and sponsors. Due to the secrecy of some of these projects Mr. Flanagan was unable to give specifics of what was being researched.

Ski Research in Austria

One of the most influential people in the ski research world is Dr. Erich Müller, who is the head of the sport science program at the University of Salzburg. He is also the chair of the Christian Doppler Laboratory "Biomechanics in Alpine Skiing" at the university. The main focus of the work that professor Müller is doing at the CD Laboratory is the biomechanical movements of the skier as he or she skis. And the work of the laboratory isn't limited to alpine skiing but also involves cross country skiing and snowboarding. Some of the recent projects have also focused on the physical demands of skiing on different groups of people, such as racers and the elderly.

The unique thing about the CD Laboratory is that it is a laboratory devoted entirely to skiing. In other countries individuals might be doing work with skiing at a university that specializes in sport science but not specifically skiing. Because of this the equipment that people have access to is designed for use with other athletes and is sometimes ineffective at measuring the movements that skiers make while skiing. These individuals are also often on their own with their projects as other people around them might not have the skiing knowledge, or desire to help these individuals with their projects. But at the CD Laboratory there are over ten highly motivated individuals working with skiing related projects who can help each other with their research. This provides a productive research environment in which many breakthroughs in the ski world are made.

While most people are on their own to fund their ski based research projects, Erich Müller is able to obtain support from partner companies to fund his research. The laboratory then is able to do a vast amount of research in various different aspects of skiing, even if the research will cost a large amount of money. The companies in turn use the research that has been completed to develop new products or modify current ones that will help set them apart from other companies.

Why Travel To Austria

Ski research isn't a high priority in the United States. Few people are devoted to conducting it and the funding and help for the research is minimal. For this reason many people who are involved with research in skiing travel to European universities that have established research facilities for conducting ski research. At the University of Salzburg CD laboratory there are currently individuals from Austria, Japan, Switzerland, and the United States, all performing research in skiing. They have come to the University of Salzburg CD Labor because it provides them with the best environment in which to conduct their research, surrounded by others who are doing similar research. While I was there I was working next to people working on various problems in skiing. For instance one individual was researching the benefits of structured teaching on the racing start on young ski racers (Kröll 2010). I was also witness to the work being done through the University of Salzburg by the FIS on preventing injuries on the world cup.

Through my travel to the University of Salzburg I received input on my numerical model and had the opportunity to work with Jörg Spörri, who is doing research similar to mine but using human subjects rather than numerical models to determine the effects of line on ski racing. While modeling his data I was able to gain information that helps me understand the line problem and application to my model.

ICSS

The International Conference on Science and Skiing (ICSS) is a gathering of the most prominent members of the skiing research community to discuss what they have been working on for the previous years. The 5th Congress took place from December 14-19 of 2010. At the conference were Jörg Spörri and Robert Reid, who are both working on similar topics as what this project covers. Attending the conference gave me a chance to see the presentations of these two individuals so that I could alter my approach to solving my problem. I was also offered the opportunity to present my work at the conference. This was an opportunity not to be passed up because while I was there I received valuable input into the work that I had completed.

Methods

Forces Acting on the Skier

Though there are many forces acting on a ski racer, not all of them are relevant to this study. It was important to identify which forces would be affected by varying lines. The first force that I could identify is that of gravity, skiing after all is the harnessing of gravity to travel from point A to point B. Because skiing takes place on earth, and I am not concerned with the effect of elevation on the gravitational pull on the skier, I used a constant 9.81 m/s^2 as the gravitational acceleration that will be acting on the skier. Because a skier doesn't often travel directly down the fall line there are two angles which effect the acceleration of the skier due to gravity. The angle alpha (α) I used to denote the slope of the hill to the imaginary flat ground plane, and the angle beta (β) I used to denote the angle describing the vector of the skier's trajectory to the imaginary line parallel to the hill but perpendicular to the fall line. The acceleration due to gravity can then be described in the equation below which is consistent with the work of David Lind and Scott Sanders (Lind and Sanders 2004).

$$a_{gravity} = g * \sin(\alpha) * \sin(\beta)$$

Equation 1: Acceleration due to Gravity

Because the imaginary skier in my model will not be traveling at a constant speed, and speed is one of the things I wish to compute, air drag becomes a factor in the model. Air drag can be represented by the following equation:

$$a_{drag} = \frac{1}{2 * m} * C * A * \rho * v^2$$

Equation 2: Acceleration due to Air Drag

When computing air drag, I am assuming that there is no wind so that the relative velocity to the surrounding air is simply the velocity of the skier relative to the ground. The variable *m* in the equation

symbolizes the skier mass which I am assuming to be 95kg. For the drag coefficient (*C*), area (*A*), and air density (ρ) I am assuming values of 0.5, 0.4 m^2, and 1.2 kg/m^3 respectively(Lind and Sanders, 2004).

As the ski slides over the snow there are friction forces that work to slow the skier down. The acceleration of the skier due to friction can be characterized by the equation below.

$$a_{friction} = \frac{F_n * \mu}{m}$$

One assumption that I am making is that the reaction force that friction acts on is only composed of the force of gravity and neglects centripetal force. In a real situation both gravity and centripetal force have a part to play in the friction on the skier. The friction coefficient, μ, that I am assuming is 0.04 (Lind and Sanders, 2004). Because of my assumptions it might seem trivial to include friction into the model, but I decided to include it for future work with the model.

The previous forces are all well known and well documented forces. What I want to include in my model is the energy that a ski racer dissipates while making a turn. Because there has been little done with energy dissipation and turn radius the equation that I picked for this is widely theoretical. It has been speculated by Dr. Brown that as one makes a tighter radius turn the amount of energy dissipated increases. As one's speed increases the amount of energy that one dissipates at a given will also increase. Different skiers will also have different dissipation curves based on their skill level. For this reason, I chose to have a user defined variable which would alter the energy dissipated per a given velocity and radius, which shall be known as the "skier ability coefficient". The higher this variable the less energy will be dissipated at a given radius and speed. I decided that for my model I would assign specific values to the skier ability coefficient for a novice, intermediate, and advanced skier; these were 7, 10, and 15 respectively. In the below equation *v* is the velocity of the skier and *d* is the skier ability coefficient.

$$a_{rad} = \frac{4}{\left(radius - \frac{v}{d}\right)} + \frac{v}{d}$$

The radius that the skier skis can be determined from any three points along the line. In this way the radius of the line does not have to be constant but can be changing throughout the course. In order to calculate the radius one needs three points with x and y coordinates for each point. With these coordinates vectors can be drawn between points 1 and 2 (P1) and 1 and 3 (P$_2$). Then the radius can be calculated from the following equations

$$\vec{P_1} = \left((x_2 - x_1), (y_2 - y_1)\right) \quad \vec{P_2} = \left((x_3 - x_1), (y_3 - y_1)\right)$$

$$h = \frac{|P_1 \times P_2|}{|P_2|} \qquad l = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$radius = \frac{\left(\frac{l}{2}\right)^2 + h^2}{2 * h}$$

Defining the Course

Not all hills are the same, some are steeper, some have rolls, and some have fall-aways. For this project I assumed the hill is exactly planar. This will make future computations much easier to perform and inserting hill characteristics into the data will not help shed any light on the question at hand. For the slope of the hill I assumed a constant angle to the ground plane. Because there are often multiple slopes that a skier must ski on during a race two slopes were analyzed, one steeper angle of 25 degrees, and one flat angle measuring 10 degrees. These are two angles that a skier may encounter in the "steeps" and "flats" sections of a course.

Ski racing encompasses four different disciplines with characteristically different turning radii. The largest turning radii occur in downhill, next super giant slalom, then giant slalom, and slalom with the smallest turning radii. For this project we will focus on slalom.

Not all slalom courses are the same. The distance between the gates can change depending on who sets the course. Distances from gate to gate may change throughout the course as well. The horizontal offset of the gates are also up to the course setters' discretion. For this project, I am assuming that every gate is the same distance from the previous gate, and that every gate has the same horizontal offset. For the Gate to gate distance I chose 9 meters as a constant value, which fits into the current regulations of 6 to 13 meters as found in USSA's competition guide (USSA 2009). For the horizontal offset I created two different courses, one with a 5 meter offset and one with a 6 meter offset. This is also within USSA's guidelines of 4 to 6 meter offset. No matter what line the skier takes down the course he will start and end at the same point.

Determination of the Line

To compute the different accelerations throughout the skiers line the line must be broken up into segments. When the line is broken up into segments the initial step size was set at 20 cm, so every 20 cm along the line a point would be placed. But I found this caused too much scatter in the results from the model, so I changed the step size to 5 cm and had much better results. This shows that as the step size approaches zero the scatter of the date will approach zero.

The skier's skis pass exactly 30 cm from the gate on every trial, which means that the center point of the turning radius changes location with every size turning radius. The skier also starts skiing and ends skiing at the same point no matter the turning radius. The only parameter that was changed between the trials was the turning radius, so any tends in the results we can be sure are from a change in the radius and not some other change in the course.

To obtain a clear picture of how the speed and time vary with the radius that the skier skis, there must be a large amount of lines analyzed. By analyzing more lines, trends can be seen through a more continuous data set. Ideally, there would be an infinite number of radii analyzed such that for any given radius there is a know time and speed. However, the process of producing the line for analysis is time consuming, so it is not possible to evaluate every possible line through the course. It was decided that radii should be analyzed from 1.5 to 3 meters and there should be a 0.05m change in radius in every line. This is a fair compromise between minimizing time gathering data to be analyzed and producing the most continuous data possible.
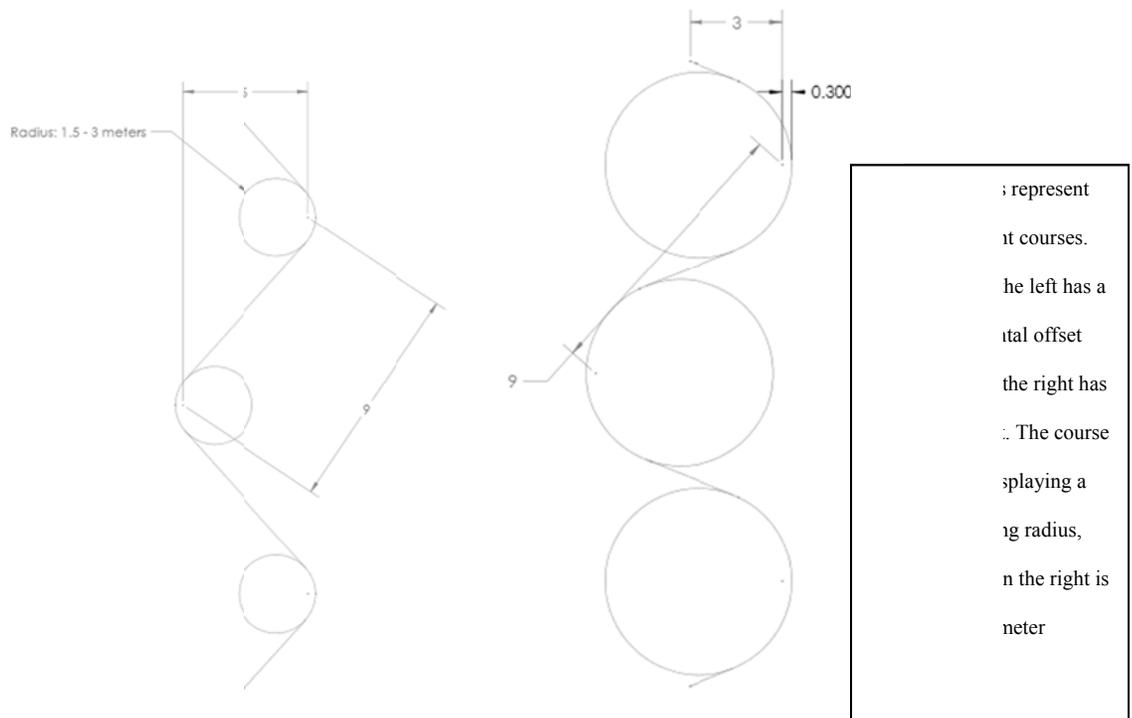

Figure 1: Drawings of Course and Line

Collecting the Line Data

It is possible to physically place a point on the line every 5 cm and record the coordinates of each point but this is time consuming and leaves a lot of room for human error. The more efficient way of collecting the data is to devise a method that computes all of the points of the line from a few key points input by the user. I created an excel spread sheet that would create the line segments based on equations

for lines and circles. The values input into the program are the center points of each circle, the slope of one of the straight segments (only one slope is needed because all of the straight segments have the same slope), the coordinates of the start and end of each line, the angle between the points on the circle that make up the turn, and at what angle relative to the fall line the turn starts. From this information points were created following the lines and circles that were 5cm apart. The only human controlled part of this method was ensuring that the transitions between lines and circles were smooth ones.

Creating the Numerical Model

A numerical model was created from the equations listed earlier in this report and Matlab was used for the computation. The numerical model created requires user inputs for gravity, drag coefficient, skier surface area, air density, coefficient of kinetic friction, mass, skier ability, slope angle, and initial velocity. By assigning user defined numbers to these variables it allows one to alter these to meet their specific needs.

The first step that the model takes is to compute the distance between all of the points, using the distance formula, and record these values into a matrix. These values will be used for later computations. The model then enters into a loop in which the radius, velocity, acceleration due to gravity, acceleration due to air drag, acceleration due to friction, and acceleration due to air drag are all calculated. All of these variables are written into individual matrices for later use and viewing. These variables also go through a degree of filtering. For instance, anomalies in the radius are sometime seen where the straight line segments meet the curved turns in the line. So a filter is placed in the model to filter out all radii that are smaller than theoretically possible, this helps alleviate roughness in the data. The total acceleration, which is simply the summation of all the accelerations, is also computed in this loop.

The next loop that the model enters is one that calculates the time for all of the different segments. These times are written into a matrix where they are stored for later computations. After this loop is finished the total time, final speed, average speed, and total distance are written into one line of a matrix entitled "data".

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All of the steps above are then repeated for all of the data sets writing the total time, final speed, average speed, and total distance into a new line into the data matrix. This matrix can be viewed and the results from all of the different lines are visible at once.
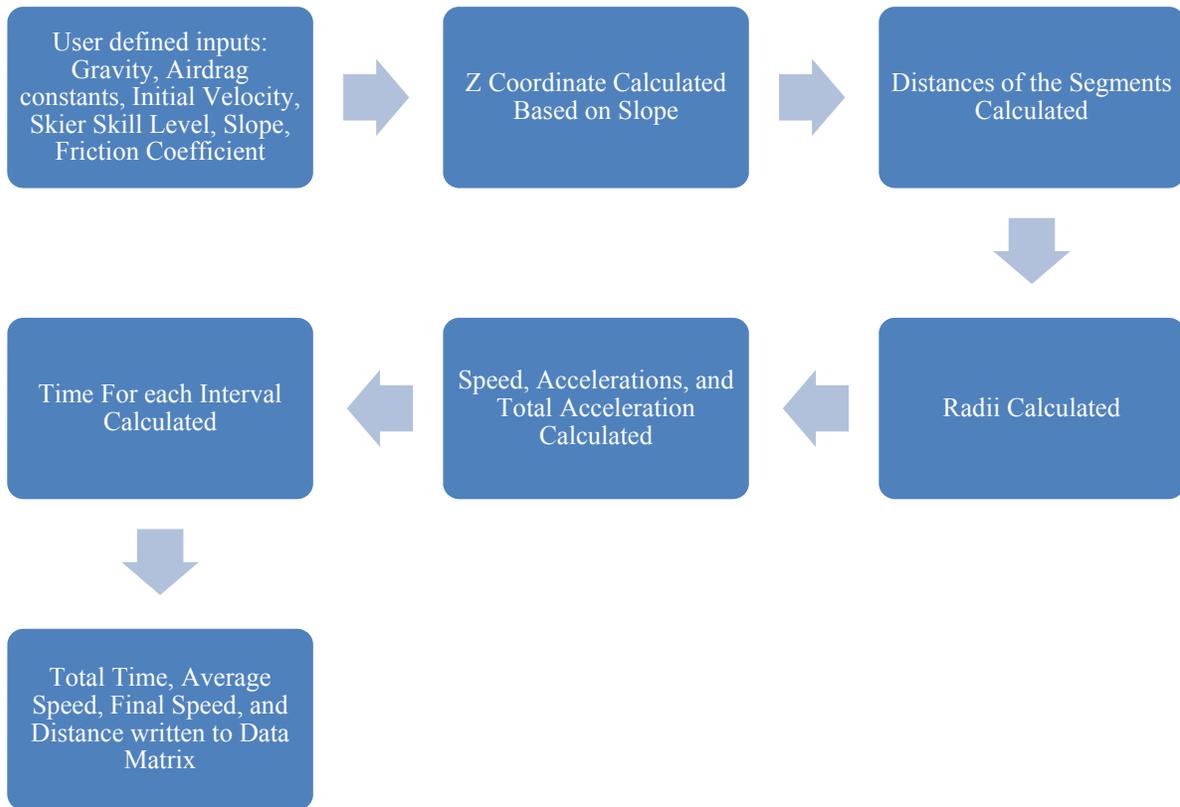

Figure 2: Flowchart of Numerical Model

Determining Entrance Speed

Because the fastest line through a course might be dependent on the initial speed of the skier, the initial velocity should be vaReid throughout the test. I decided to do analysis on three different speeds, 0, 7 and 10 m/s. For analysis on the flat section I chose to assume a 10 m/s entrance speed. This is because the flats section of a course is usually at the middle or end of the course, so at this point the skier already has a velocity entering the section. It is rare that a course would start with a flat section for a skier to navigate so a standstill start was not analyzed.

Display of Results

For easy analysis and interpretation of the data, charts are to be created so that the trends of speed and time can be seen as the turning radius changes. Three different charts will be made for each course at each initial velocity tested. One chart was created for time for the course completion against the radius that the skier skis. This chart is the most important for answering the question of which line is faster because the winner of the ski race is the one who makes it to the bottom first, not the one with the highest speed. Another chart was made displaying the average speed against the radius. This chart is more to show that the fastest time through the course is not necessarily produced from the line that produced the highest average speed. The third chart created has the final speed against the radius. This chart is important since there are situations in a race where the final speed coming out of a section is more important than skiing that particular section as fast as possible. An example of this is if a skier was about to enter a flat section of the course. At this point a skier would want to have the highest possible speed coming onto the flats. On all of these charts the data from a novice, intermediate and advanced skier will be displayed at once so that comparisons can be made.

Results

Step Size

Originally a 20cm step size was selected for the determination of points along the skied line. When these points were analyzed the results showed a significant amount of scatter as can be seen in the figure below.
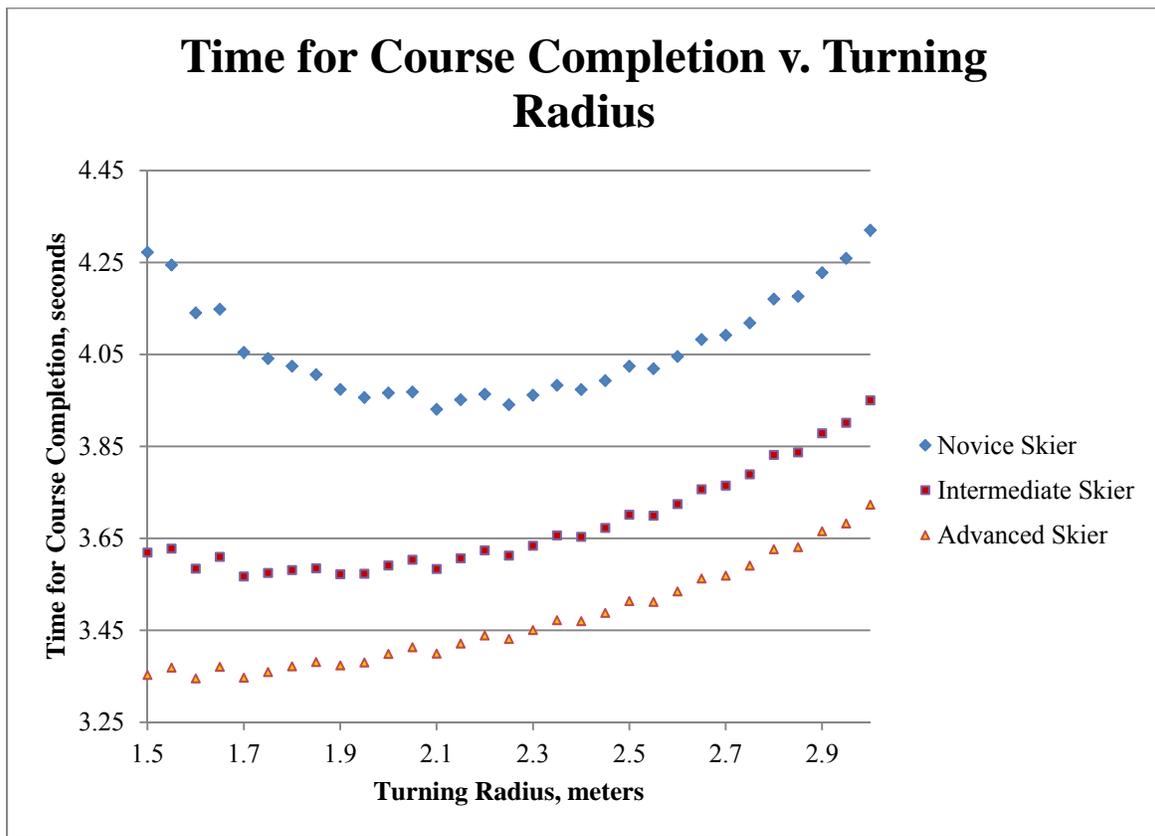
Figure 3: Scatter at a 20cm Step Size

Through much trial and error it was determined that the step size was the cause of this scatter. When the step size was altered to 5cm the data began to fall towards a more unified pattern. Though there was still some scatter in the results it was far less than with the 20cm step size and it was determined that it was consistent enough to draw conclusions from. The figure below displays the minimal scatter seen at the 5cm step size.
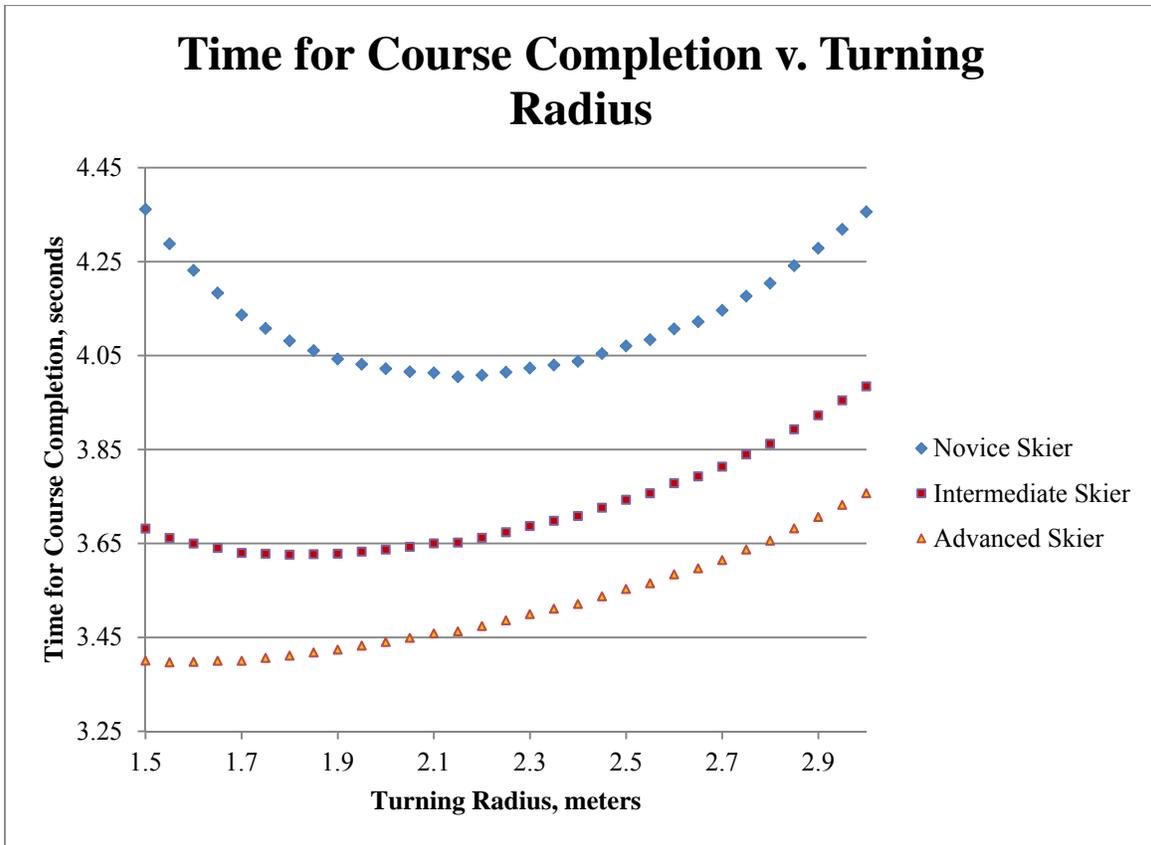
Figure 4: Scatter at a 5cm Step Size

Stand Still Start Run Time

When starting at a standstill it is clear that it is faster to go straighter at the gates than taking a more rounded line. In the course with a 6 meter gate offset all three skiers saw their fastest time at a radius close to the smallest radius analyzed. So if one were to choose a line for the first few gates of an offset course it is best for one to go as straight as possible at the gates.
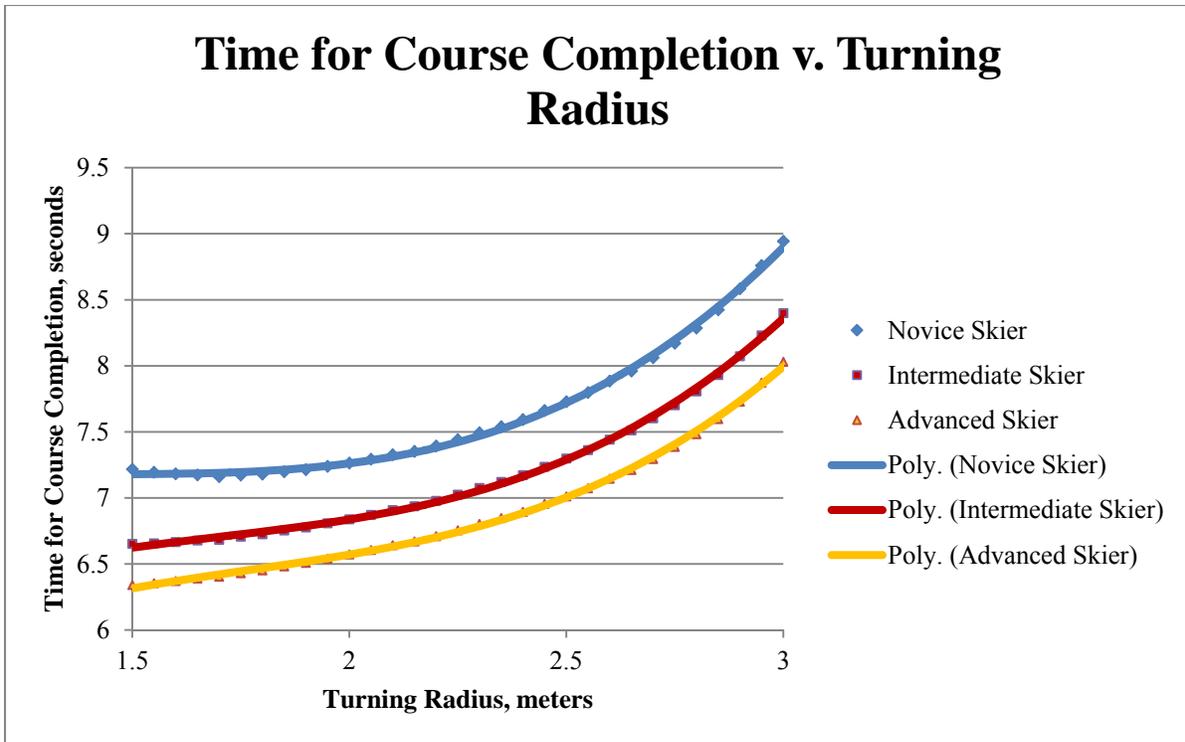
Figure 5: Time for Completion (Standstill Start, 6m offset)

When the course is less offset the results are less clear cut. While the intermediate and advanced skiers both saw their fastest times at the smallest radius analyzed, the Novice skier saw his fastest time at a turning radius around 2 meters.
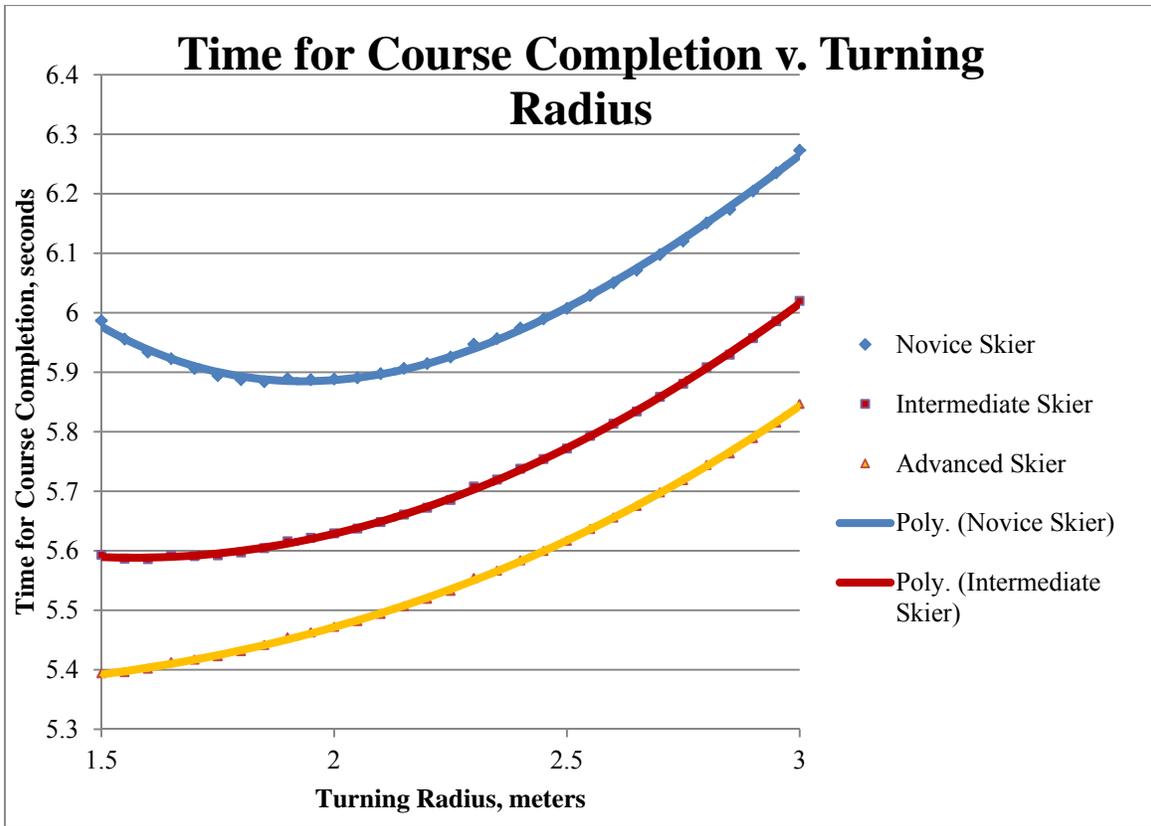
Figure 6: Time for Completion (Standstill Start, 5m offset)

Skier Type	Optimal Radius 5m	Optimal Radius 6m
Novice	1.85	1.7
Intermediate	1.55	1.5
Advanced	1.5	1.5

Table 1: Optimal Radii for Standstill Start (time)

7 m/s Run Time

At a speed of 7 m/s both the intermediate skier and the novice skier saw their fastest times at a radius between the minimum and maximum radii tested, while the advance skier saw his fastest time at the smallest radius analyzed on both courses. The curve traced by the time data is also not a symmetrical one for the novice and intermediate skier. As they ski a smaller radius than the optimal radius time their time increases more rapidly than if they had skied a line with a larger radius that the optimal radius. This shows that a skier might be able to decrease his time by skiing a more direct line to a point, but once he is beyond that point his time could be worse than if he had skied a line an equal amount wider from the optimal line.
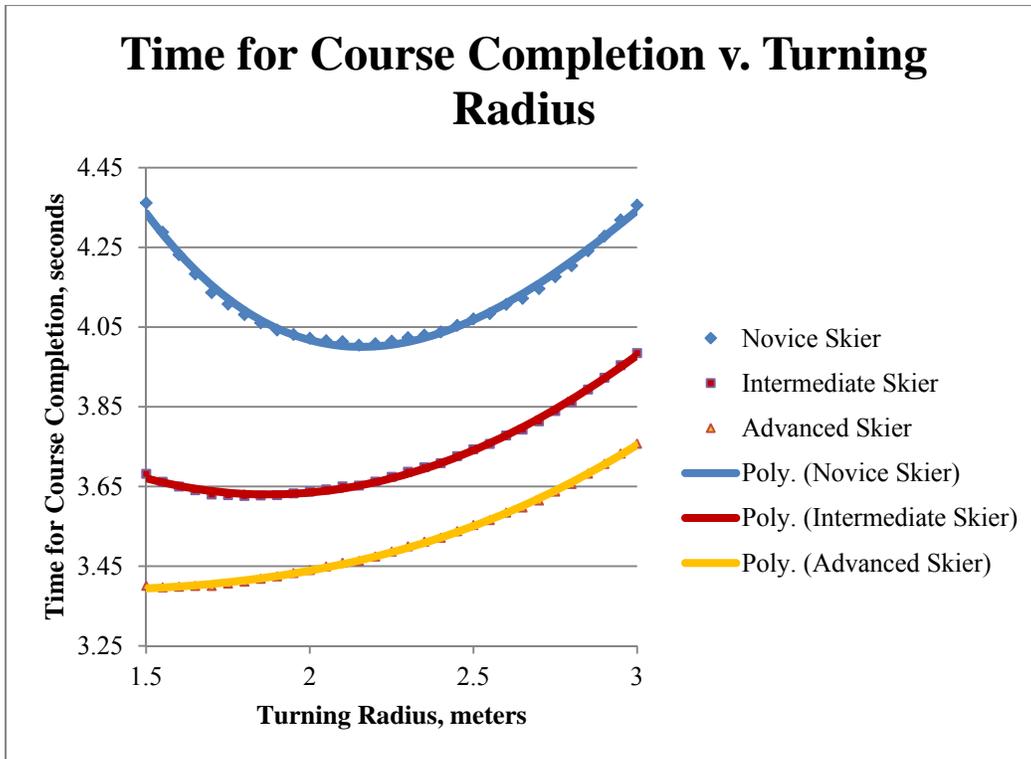
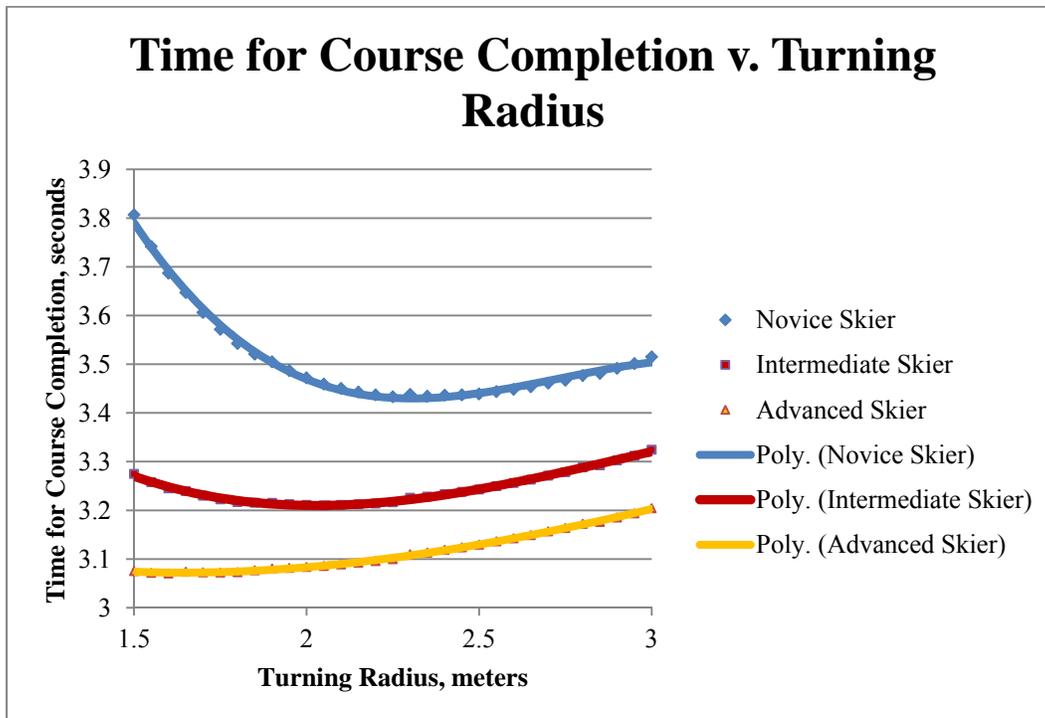Figure 7: Time for Completion (7 m/s start, 6m offset)

Figure 8: Time for Completion (7 m/s start, 5m offset)

Skier Type	Optimal Radius 5m	Optimal Radius 6m

Novice	2.25	2.15
Intermediate	2.05	1.8
Advanced	1.6	1.5

<div align="center">**Table 2: Optimal Radii for a 7m/s Start (Time)**</div>

10 m/s Run Time

At a speed of 10 m/s the results look similar to that of the 7 m/s run. The advanced skier still has his/her fastest run close to the smallest radius analyzed, and the intermediate and novice skiers still have their fastest runs at a radius between the maximum and minimum radii. The same asymmetrical curve characteristics can be observed in the novice and intermediate skier's data. What is different here is that in the 5m offset course the novice skier sees almost no gain by taking a line with a smaller radius than the maximum radius analyzed. But once the novice skier goes below the optimal radius his time begins to increase dramatically.
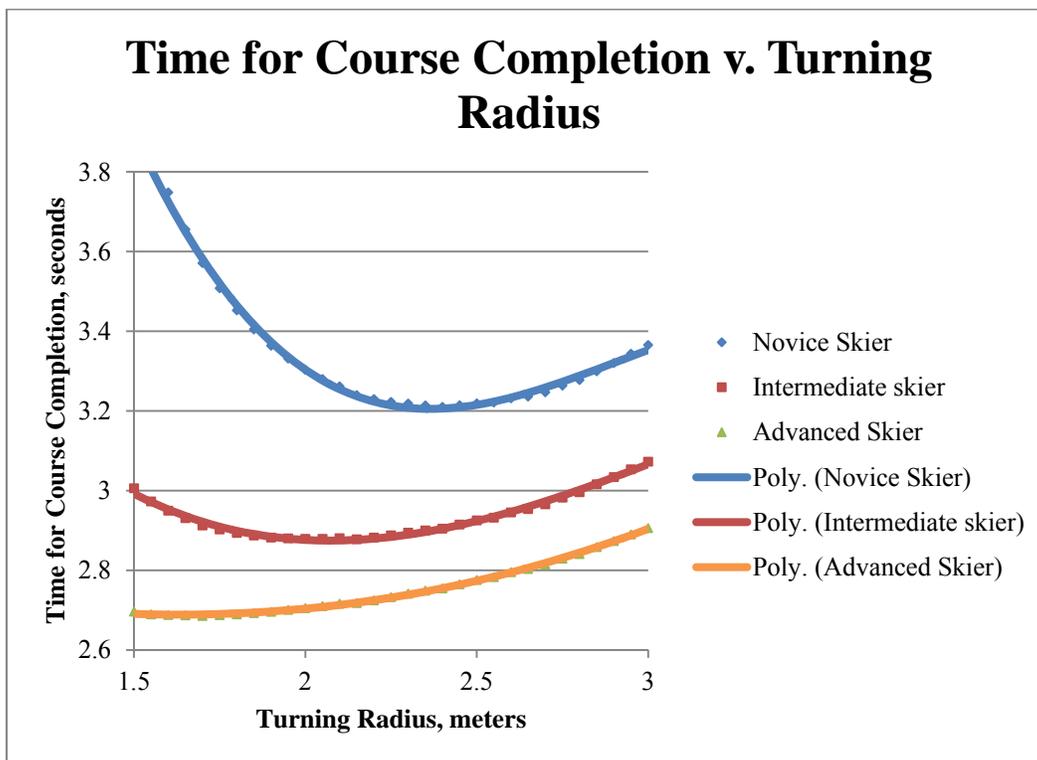
<div align="center">**Figure 9: Time for Completion (10 m/s start, 6m offset)**</div>

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Time for Course Completion v. Turning Radius
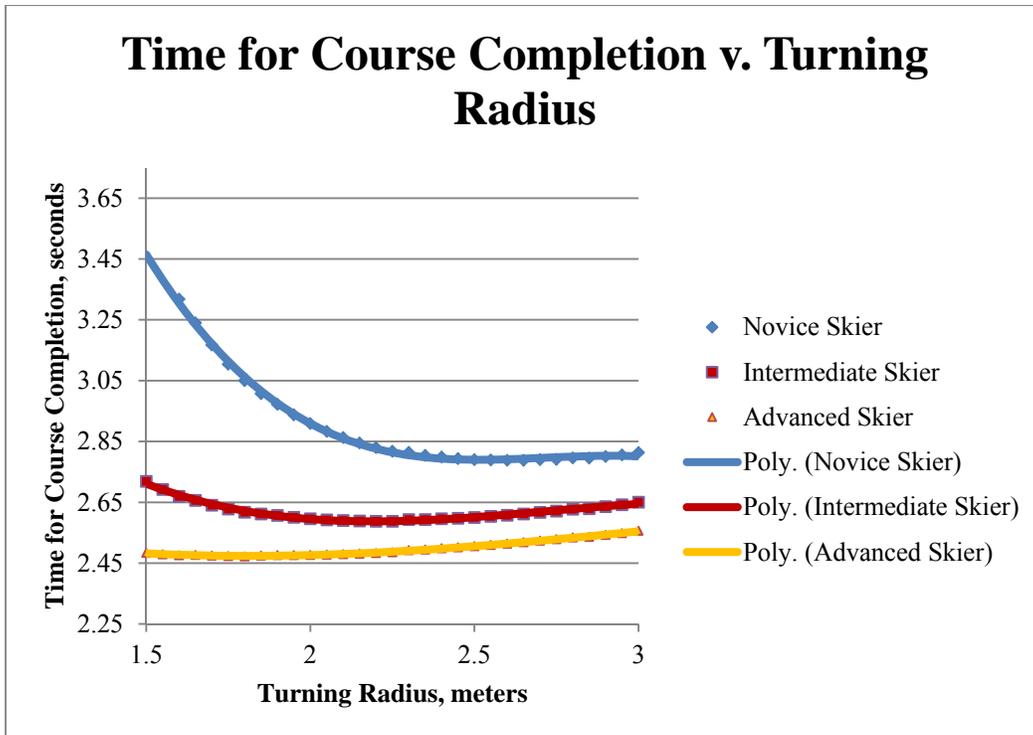
Figure 10: Time for Completion (10 m/s start, 5m offset)

Skier Type	Optimal Radius 5m	Optimal Radius 6m
Novice	2.65	2.4
Intermediate	2.25	2
Advanced	1.8	1.7

Table 3: Optimal Radii for 10m/s start (Time)

How Optimal Radius varies with Speed

The optimal radius increases with increasing speed. This can be seen in the trails below which took place with a 5m offset with the steep slope angle.

Optimal Radius v. Initial Speed

Figure 11: Optimal Radius and Speed (5m offset, steeps)

Average Speed and Run Time

What is interesting to note is that when the skier is traveling at his fastest average speed through the course, he is not necessarily having his best run. This is highlighted in the two figures below. The novice skier obtains his highest average speed at a turning radius of 2.35m, while his fastest run time occurs at 2.15m. This showcases that the shorter traveled distance of the 2.15m course is enough to offset the slower average speed seen here versus the speed seen at a radius of 2.35m. Other instances of this occurrence can be seen throughout the data.

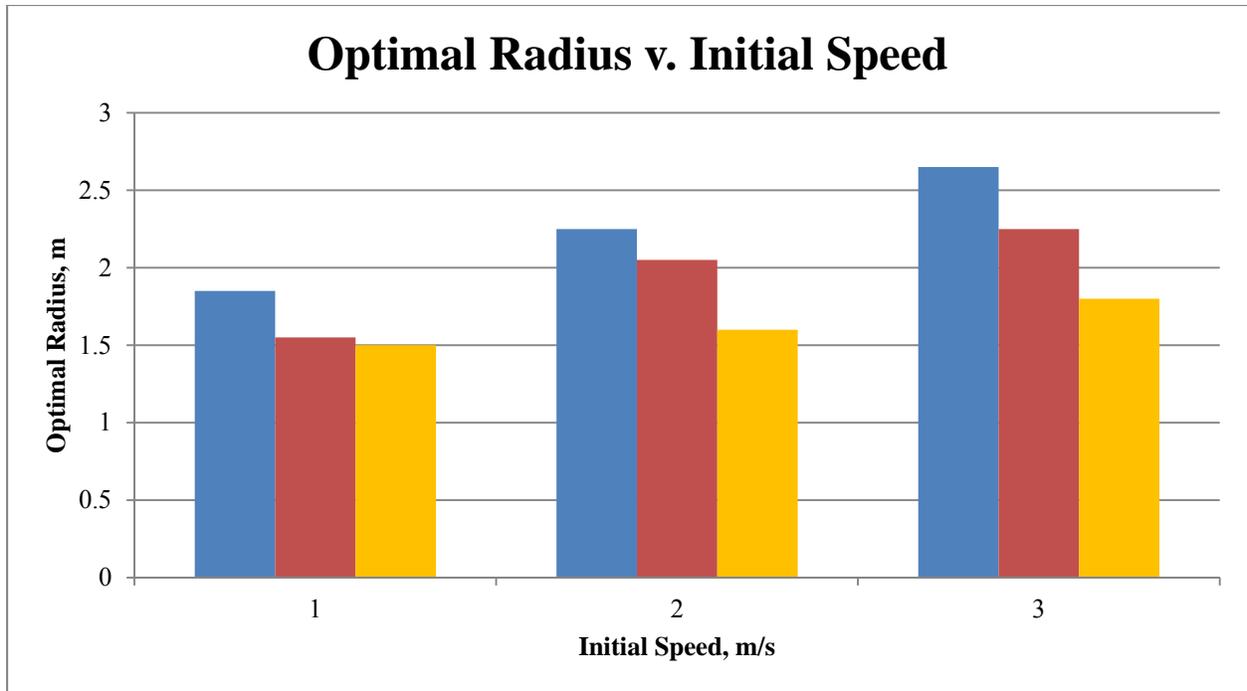| Trial	Optimal Radius for Average Speed, m	Optimal Radius for Time, m
Novice skier, 7m/s, 6m offset	2.35	2.15

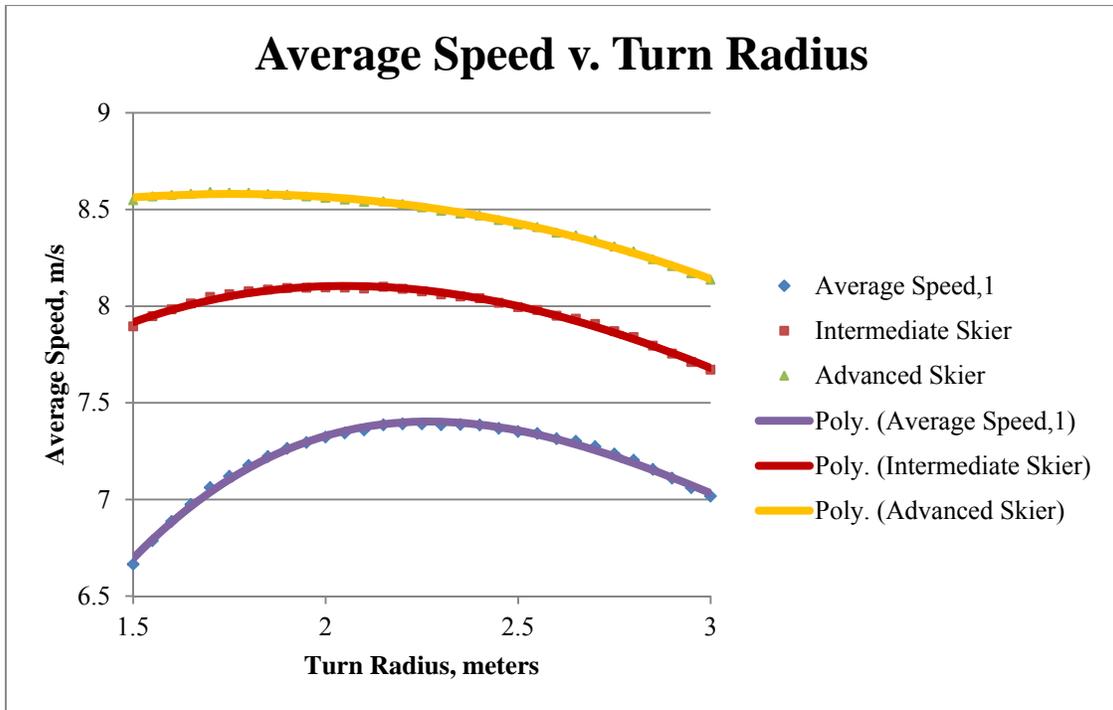Table 4: Comparison of Radii for Optimal Average speed and Optimal Time

Figure 12: Average Speed (7 m/s start, 6m offset)
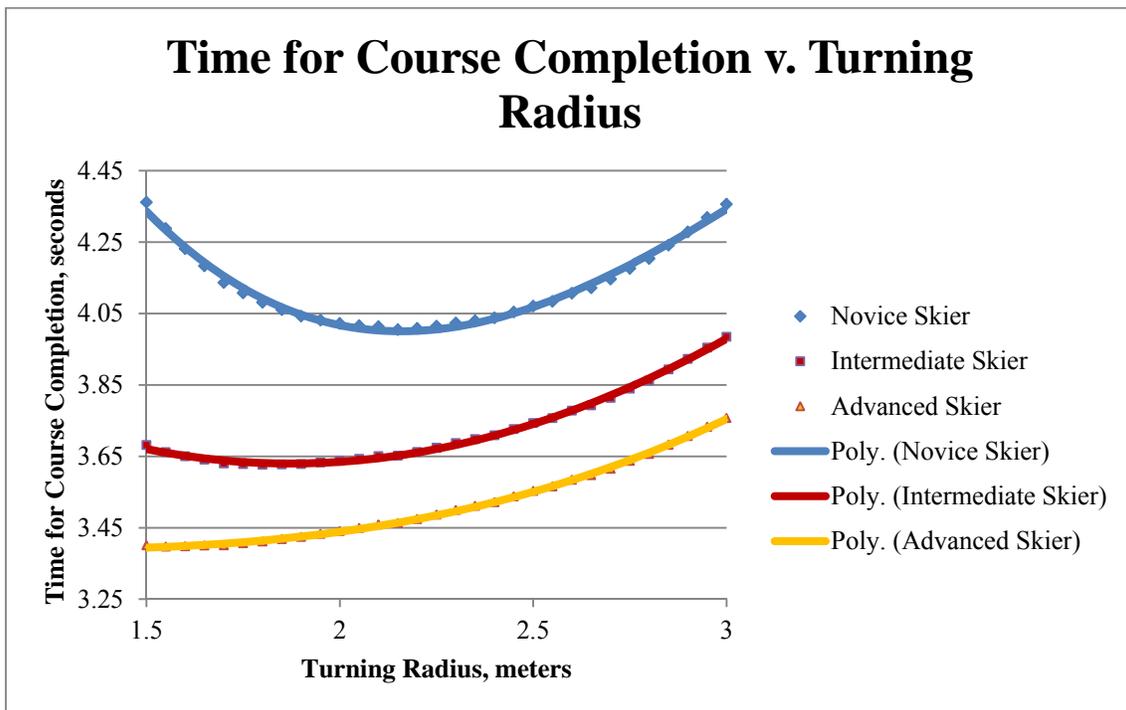
Figure 13: Time for Completion (7m/s start, 6m offset)

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Maximizing Final Speed

When entering a long flat section of a course it is thought best to maximize one's speed to ensure a fast time through the flats, even if it means a slower time through the few gates beforehand. Similar to average speed, the highest final speed occurs at a radius larger than that which produces a minimum course time. The final speed of the skiers when starting at 7 m/s and with a 6m offset can be seen below. The radius which produces the fastest time is 2.15m while the radius that produces the highest final speed is 2.35m.

Trial	Optimal Radius for Time, m	Optimal Radius for Final Speed, m
Novice Skier, 7m/s, 6m offset	2.15	2.35

Table 5: Comparison of Optimal Radii for Time and Exit Speed
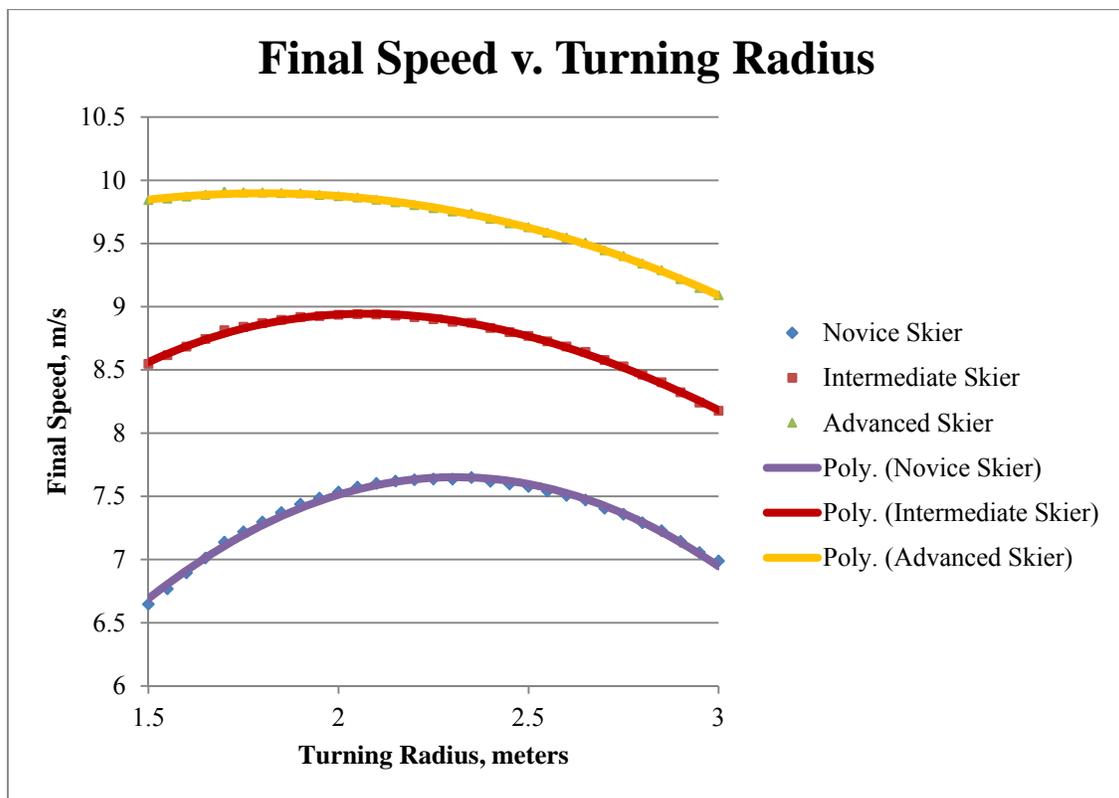
Figure 14: Final Speed (7 m/s start, 6m offset)

Skiing on the Flats

When skiing on the flats the optimal radius for skiing with the fastest time is different than that of the steeps. On the steeps with an initial speed of 10 m/s and a course offset of 5m the optimal radius occurred at 2.65m for the novice skier. While with the same course and initial speed but skiing on the flats the optimal radius was 2.55m. The same trend was seen on the 6m course offset there the optimal radii for the steeps and flats were 2.4m and 2.25m respectively.

Skier Skill and Offset	Optimal Radius for the Steeps	Optimal Radius for the Flats
Novice, 5m	2.65	2.55
Novice, 6m	2.4	2.25

Table 6: Comparison of Optimal Radii on the Steeps and Flats

Figure 15: Time on the Flats (5m offset)
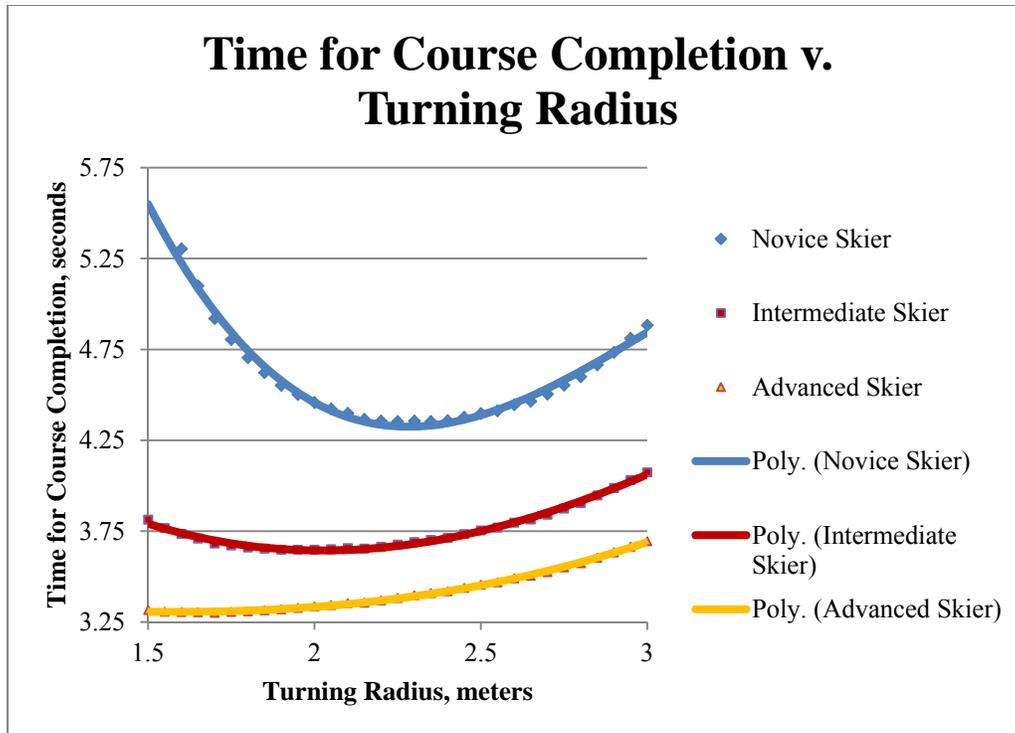
Figure 16: Time on the Flats (6m offset)

Asymmetrical Relationship between Radius and Time

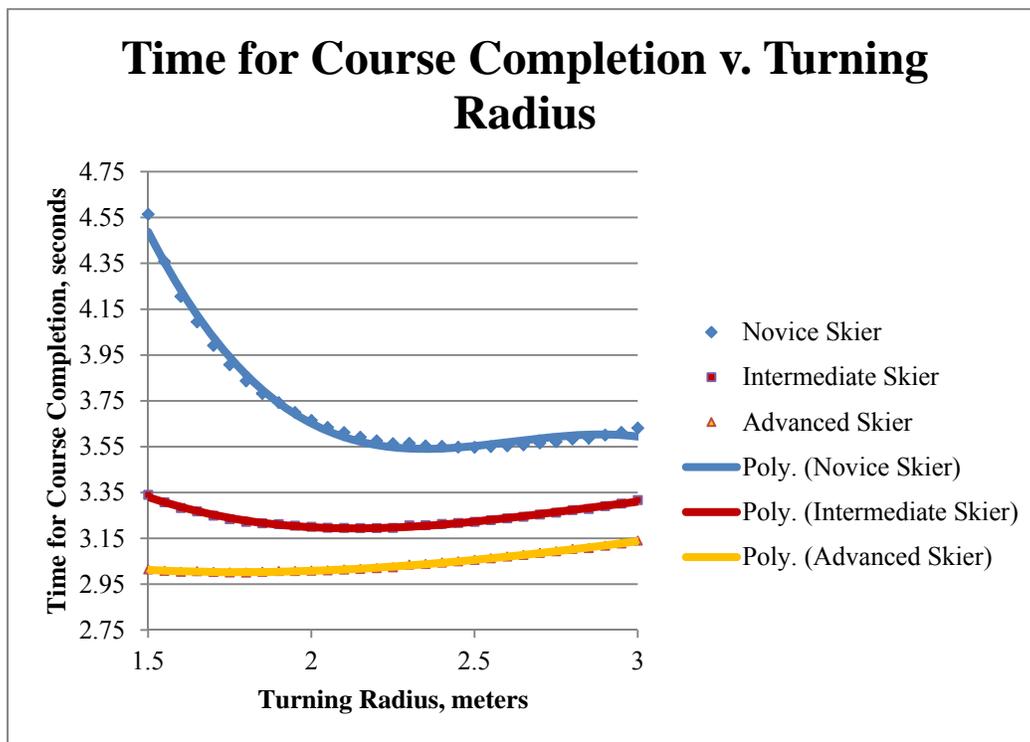The relationship between radius and time is not symmetrical about the optimal radius. If one were to look at any of the above figures he would see that it is often the case that as the radius goes below the optimal radius the time increase more rapidly than when the radius is increasing from the optimal radius. The table below shows how the time is effected by the radius on a 7m/s trial on the steep by the novice skier.

Radius	Time, seconds	Difference in Time From Optimal, seconds
1.5	3.806	0.372
2.25 (optimal)	3.433	0
3	3.515	0.082

Table 7: Asymmetrical Change in Time

The Effect that Course Setting has on Time and Speed

From the results of the trials it is clear that the more offset the course is, the slower the skier travels and the longer it takes for him to complete the course. Using a 7m/s start speed on the steeps as an

example the novice skier was able to reach a higher maximum speed, average speed, and lower time at the

5m offset than he was able to reach at the 6m offset.

Offset	Minimum Time, s	Maximum Average Speed, m/s	Maximum Final Speed, m/s
5m	3.43	8.41	9.45
6m	4.00	7.39	7.64

Table 8: Comparison of Speed and Time Related to Course Set

Discussion

Model Validity

It is important to examine the validity of this model to determine its implications in the skiing world. This is a numerical model, and being a numerical model, there is a high degree of validity within the model. There are no outside parameters acting on the data, so all results output by the model can be seen as directly related to the forces that the model subjects the data to. Because of this we can say with certainty that the results are directly related to the inputs. However, because this is a numerical model and there was no human being involved in any stage of this project, no specific numerical predictions on what would happen in real life and be made from this model. The energy dissipation equation for instance is a purely theoretical curve which was not based on any measurements done on a skier, a real skier might have a energy dissipation curve that looks different from the one assumed. These ideas of model validity were discussed in a trial lecture give by Robert Reid before his doctoral defense (Reid 2010).

The accuracy of the results from the model are only as accurate as the data input into the model. A 5cm section length was utilized when defining the line to save time while still maintaining a good deal of accuracy in the line definition. However, a smaller section length would produce an even more accurate result. The increment in radius size is also another source of error in the results. A 5cm increment was used in increasing the radius of the line the skier skied. Because of this we can only theorize what is occurring between these individual radii analyzed. This results in a 5cm tolerance on any specific radius cited in this report, for instance if an optimal radius was stated to be 2m it most likely somewhere between 1.95 and 2.05 and not exactly at 2m. In the future more radii should be analyzed so that a clearer picture can be seen as to what is happening at every radius between 1.5 and 3 meters.

Starting from a Standstill

When starting from a standstill it is clear from the results that it is faster to go straight at the gates. The degree of straightness depends on the offset of the course and the ability level of the skier. As

one can see from figure 5 when the course is at the maximum horizontal offset, it is faster no matter what the skier's ability level to go straight at the gates. When the course is less offset it is not as clear cut. In figure 6 one can see that when the offset is only 5 meters the intermediate and advanced skier benefit the most from taking a direct line at the gates, but the novice skier experiences his fastest time at 1.85m. From these results we can make a correlation between time, radius, offset, and ability level.

For a given ability level the fastest time may be achieved at a specific radius for a specific offset. As the offset is decreased the radius at which the fastest time is achieved becomes larger. Likewise, if the skill level is decreased the optimal radius is also increased.

Starting at Speed

When starting at race speed of 7 m/s and 10 m/s it is clear that both the intermediate and novice skiers have an optimal radius to complete the course in the fastest time. The advanced skier saw the fastest time when he skied the most direct line analyzed. This is true for both course offsets analyzed. The optimal radius of the novice skier was larger than that of the intermediate skier. This shows the inverse relationship between skill level and optimal radius.

In figure 11 we can see that as the starting speed increases the optimal radius increases as well. The optimal radius for the novice skier at a 7 m/s start speed on a course with a 5m horizontal offset is 2.15m, while the optimal radius for the same course with a starting speed of 10 m/s is 2.65m. A similar trend can be seen in the optimal radii for the intermediate skier. This demonstrates the direct relationship between starting speed and optimal turning radius.

Asymmetrical relationship between Radius and Time

One should notice the asymmetrical shape of the curve that the data traces on the time v. radius graphs, such as figure 9. The change in completion time with respect to change in radius is smaller as one approaches the optimal radius with a decreasing radius than an increasing radius. One is better off skiing a wider line that the optimal radius than pushing the limits and skiing below the optimal radius. If one skies

below the optimal radius they will be skiing slower than if they had skied a line with a turning radius an equal distance larger than the optimal turning radius.

Average Velocity and Fastest Course Time

As one can see from the result section and figures 12 and 13 the highest average velocity through the course does not always produce the fastest course time. The fastest course time, from what can be interpreted by the results of this project, is produced at a radius slightly smaller than the one which produces the highest average velocity. This is a perplexing problem to many coaches who coach by what they perceive as the fastest skiing, rather than by what actually produces the fastest course time. If a skier were to come through a course carrying a lot of speed a coach might be inclined to say this skier is skiing faster than someone who is skiing a tighter line. Though he may be correct that the fist skier was traveling faster relative to the snow directly underneath him, the second skier could possibly have a faster course time even though he appears to be going slower. And without on hill measurement of time this may be difficult to detect.

Maximizing Exit Velocity

If a skier were about to enter the flats it is thought to be desirable to maximize ones velocity so that he may pass over the slow flat section as quickly as possible. If one compares figure 14 and figure 7 they will see that fastest course time does not occur at the same radius as maximal exit velocity. The maximal exit velocity occurs at a radius larger than that of the optimal radius for time. So if one desired to maximize his velocity before a flat section he should take a wider line through the few gates directly before the flats.

The question behind this strategy is if this strategy will actually be beneficial to the skier. Will the time he loses in the few gates before the flats be offset by the savings on time if he had skied the section before the flats as fast as possible? These are questions that cannot be answered from the work done in this project and should be pursued in future works.

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Skiing on the flats

The current theory on skiing in the flats is that a wider line is favored over a tight line to "carry" one's speed through the entire flat section. The results of this project on the other had said otherwise. It is clear it is faster to ski a tighter line on the flats than on the steeps for an identical course set and entrance speed. One question that is left unanswered by this project is whether it is faster to sacrifice a little bit of speed on the steeps to ski the flats as fast as possible or to ski the steeps as fast as possible and take the slower time on that flats as a consequence. This will have to be answered through future experimentation.

Possible Regulation Changes

Based on these results it is clear to see that there is an optimal radius that will allow a skier to reach his maximum speed. But sometimes the speed that racers reach is unsafe, and when they crash the risk of suffering a major injury increases. The International Skiing Federation has developed several equipment and course regulations to limit the speed that competitors reach. From what we can see in the results, setting a course with more horizontal offset will result in a skier achieving a slower average speed through the course.

Another way to slow skiers down could be to regulate the equipment such that it is impossible to reach the minimum turning radius. This can be done by increasing the minimum length of the ski or the side cut of the ski. By combining equipment and course setting regulations the speed of alpine ski racers can be controlled.

Conclusion

- This model has a high degree of validity within the results, while having a low degree of validity with respect to real life

- From a stand still start it is more beneficial to go straight at the gates. Optimal radius is inversely proportional with offset and skill level.

- Optimal turning radius is inversely proportional with skill level

- Optimal turning radius is directly proportional with speed

- The relationship between time and turning radius is asymmetrical about the optimal radius

- The fastest average speed does not produce the fastest course time

- If one wishes to maximize his exit velocity he should take a slightly wider line

- A tighter line on the flats than on an equally set course in the steeps results in a faster time

- Less offset courses result in higher speeds and faster times

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Appendix A: Detailed Listing of Optimal Radii

Below is a table of all of the optimal Turing radii for every situation analyzed in the project. All radii have a tolerance of plus or minus 0.05 meters.

Course offset, meters	Initial Speed, m/s	Steep or flat	Skill Level	Optimum Radius for Time, meters	Optimum Radius For final Speed, meters	Optimum Radius for Average Velocity, meters
5	0	Steep	Novice	1.85	2.35	2.05
5	0	Steep	Intermediate	1.55	2.1	1.75
5	0	Steep	Advanced	1.5	1.85	1.55
5	7	Steep	Novice	2.25	2.5	2.5
5	7	Steep	Intermediate	2.05	2.25	2.25
5	7	Steep	Advanced	1.6	1.85	1.8
5	10	Steep	Novice	2.65	2.75	2.85
5	10	Steep	Intermediate	2.25	2.45	2.25
5	10	Steep	Advanced	1.8	2.15	2.15
5	10	Flat	Novice	2.5	2.5	2.5
5	10	Flat	Intermediate	2.2	2.2	2.25
5	10	Flat	Advanced	1.8	1.85	2.05
6	0	Steep	Novice	1.7	2.05	1.8
6	0	Steep	Intermediate	1.5	1.7	1.55
6	0	Steep	Advanced	1.5	1.5	1.5
6	7	Steep	Novice	2.15	2.35	2.25
6	7	Steep	Intermediate	1.8	2.05	2.15
6	7	Steep	Advanced	1.5	1.7	1.7
6	10	Steep	Novice	2.4	2.5	2.55
6	10	Steep	Intermediate	2	2.25	2.2
6	10	Steep	Advanced	1.7	1.9	1.9
6	10	Flat	Novice	2.25	2.25	2.4
6	10	Flat	Intermediate	1.9	2.05	2.15
6	10	Flat	Advanced	1.7	1.7	1.7

Appendix B: Interview Notes with Troy Flanagan

What type of research, if any, are USSA and the US ski team investing in at the moment? Is it more geared towards physiology, equipment, technique, and how many projects are in the works?

>Research is being conducted primarily in physiology, sports nutrition and sports technology/equipment design.

What disciplines have the most research concentrated on them?

>Alpine and Nordic Combined

Who is doing this research, is it more educational institutions, internal sports scientists, or industry?

>USSA has an internal sport science department that conducts the majority of research. We do partner with industry and universities for big projects.

How much funding is being given to these projects, and who generally receives it?

>Approx $1 million dollars over the Olympic cycle (4 yrs)

Where does this funding come from; is it from USSA or from other sponsors?

>Sponsors/donors and Universities (grants)

 Do you mainly perform research and development to fix a current problem such as injuries, or is it more to improve the quality of racers in the United States?

>More problem solving than anything

If someone were to come to you with an idea for a research project would USSA be likely to support the project if it showed potential, or are you strictly focused in the direction of your ongoing research?

>We would highly welcome the project. In fact during the Vancouver Games we ran a program to invite new ideas

How likely is it for a group at a university or a doctoral student to get funding or support for their skiing based project?

>Somewhat likely

In your experience how much of a "return on your investment" in research is there? Do you feel that most research is a waste of money and effort, or have some recent breakthroughs come about from research?

>Research is only conducted on performance and ways to enhance it. There were a number of projects prior to Vancouver that directly delivered performance enhancements.

How are new findings from research implemented in the USSA system? Do you immediately implement a new system or idea, putting full confidence in the research, or does the USSA slowly implement the system to see if it does in fact make a difference?

>We are quick to adopt new technologies

Finally can you point to any specific improvements that have come about through the research of skiing either funded or supported by the USSA, or as an independent project?
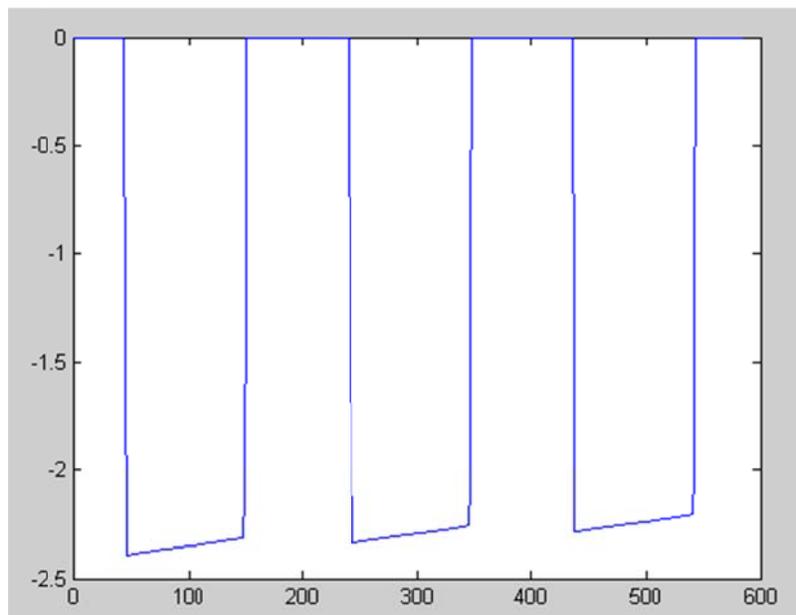
>A number of technology projects directly enhanced performance. Unfortunately, I am not permitted to make these public.
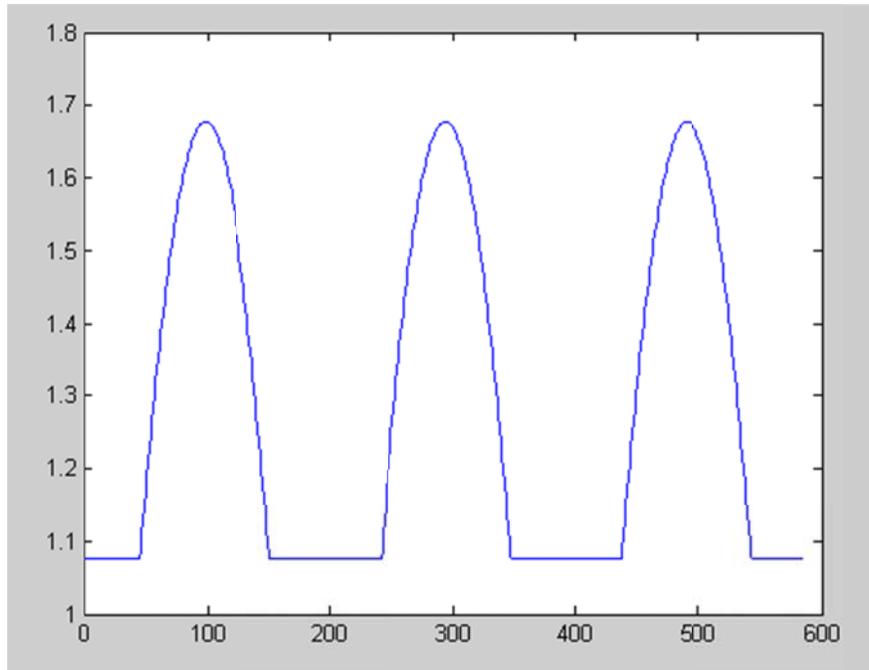
Appendix C: Accelerations throughout the Course

All of these graphs were taken from a trial on the flats with an initial velocity of 10m/s and a 3m turn radius
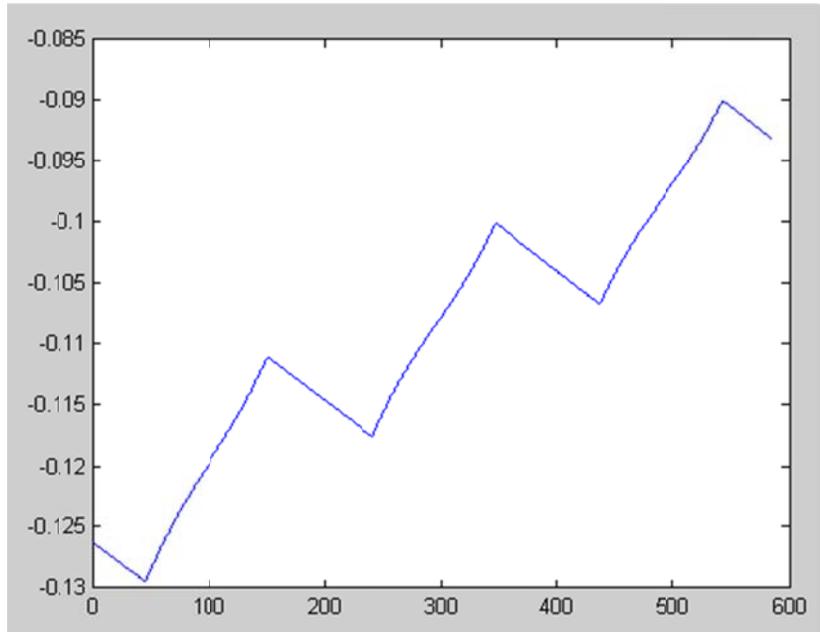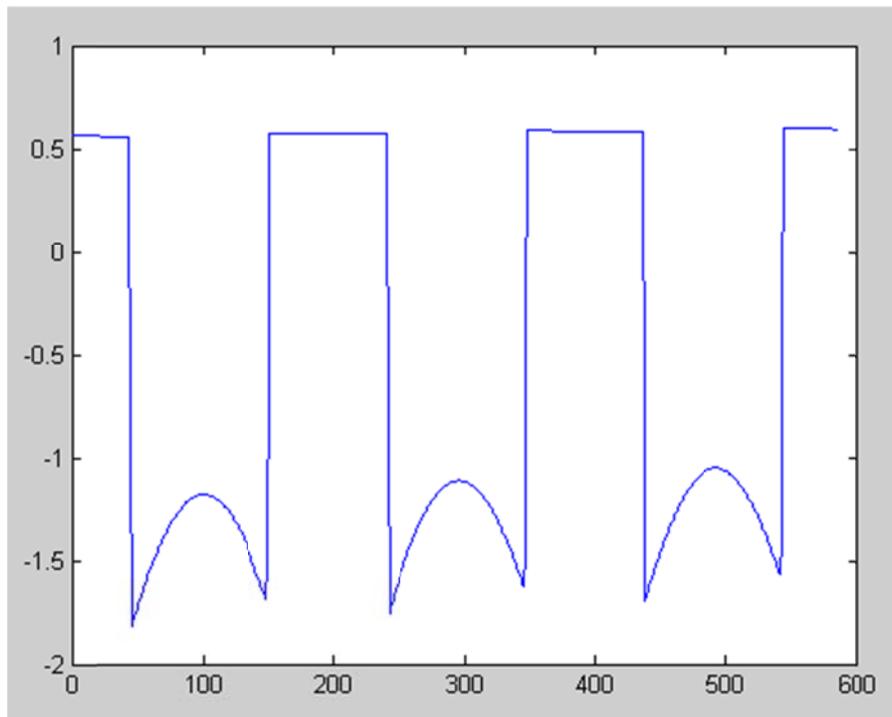

Air Drag Acceleration, m/s^2

Radius Acceleration, m/s^2

Slope Acceleration, m/s^2

Total Acceleration, m/s^2